Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

ADDITIONAL MATHEMATICS **4037/21**

Paper 2 **May/June 2023**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

DC (ST) 326647

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

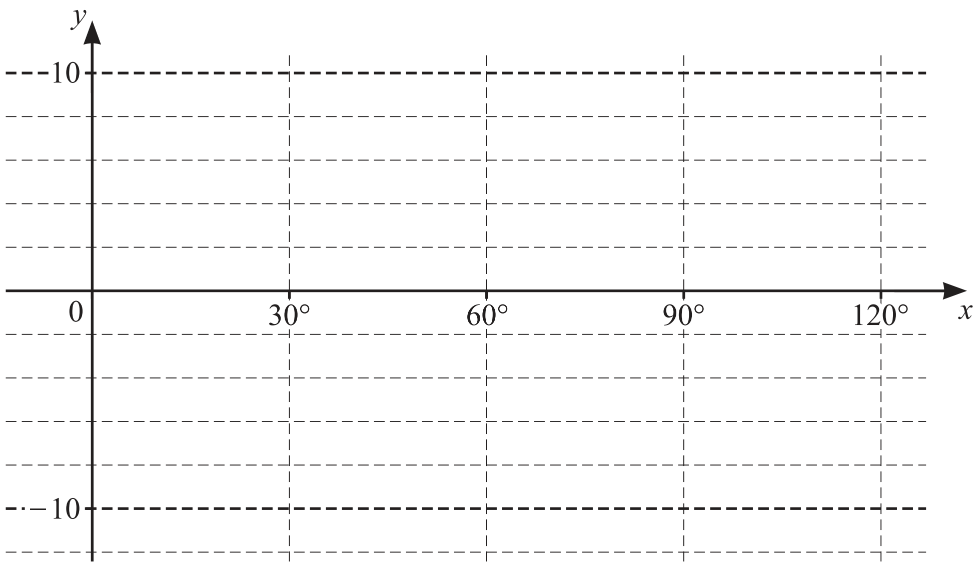
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Variables x and y are such that when $\lg y$ is plotted against $\sqrt{x}$ a straight line passing through the points (1, 5) and (2.5, 8) is obtained. Show that $y = A \times b^{\sqrt{x}}$ where A and b are constants to be found. [4]

2 The function g is defined for $0° \leqslant x \leqslant 120°$ by $g(x) = 2 + 4\cos 6x$.

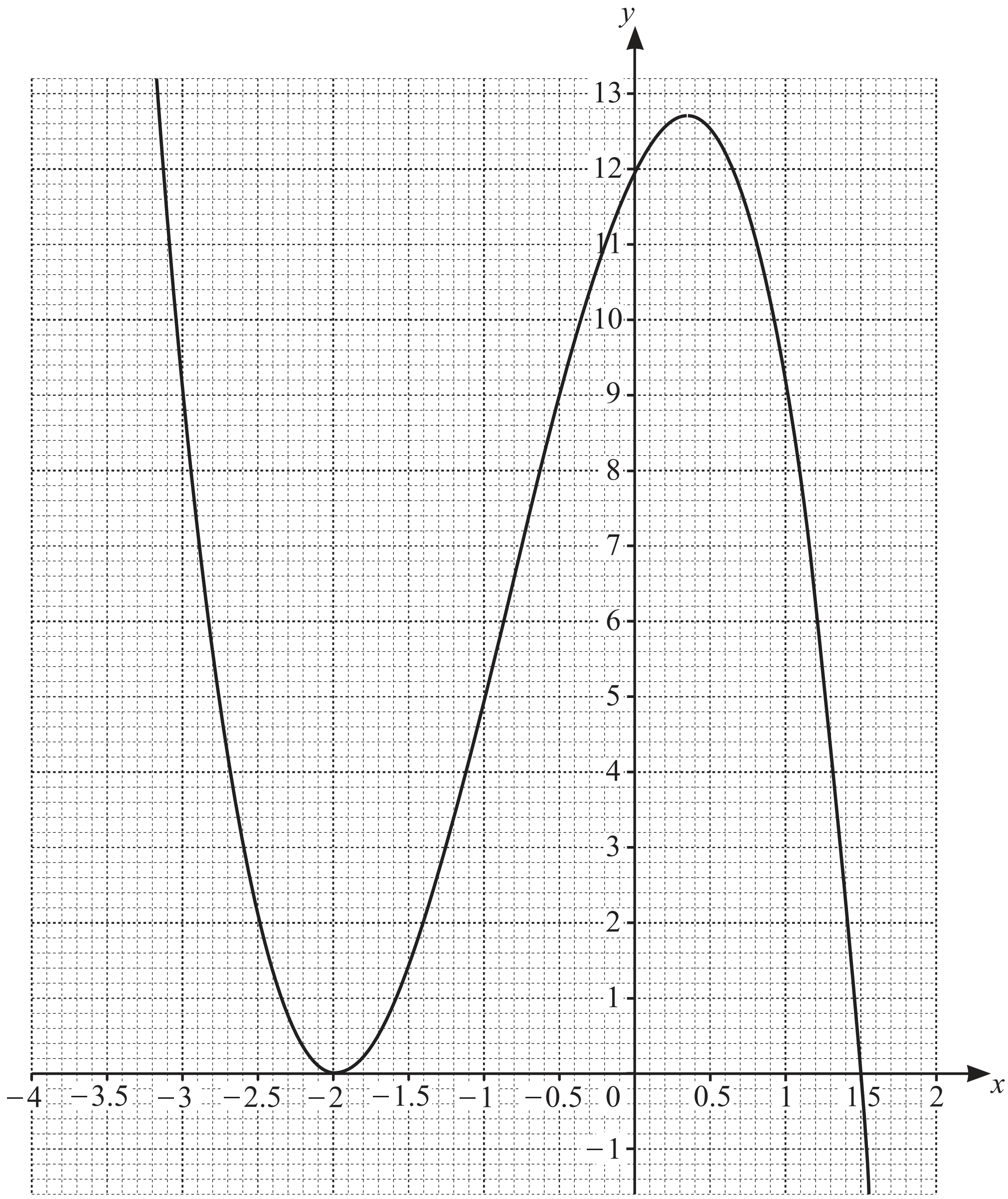
(a) On the axes, sketch the graph of $y = g(x)$. [3]



(b) State the amplitude of g. [1]

(c) State the period of g. [1]

3



The diagram shows the graph of $y = \mathrm{h}(x)$ where $\mathrm{h}(x) = (x+a)^2(b+cx)$ and a, b and c are integers. The curve meets the x-axis at the points $(-2, 0)$ and $(1.5, 0)$ and the y-axis at the point $(0, 12)$.

(a) Find the values of a, b and c. [2]

(b) Use the graph to solve the inequality $\mathrm{h}(x) \leqslant 9$. [3]

4 **(a)** Solve the equation $5^{2y-1} = 6 \times 3^{y}$, giving your answer correct to 3 decimal places. [3]

(b) Solve the equation $e^{2x} - 4 + 3e^{-2x} = 0$, giving your answers in exact form. [4]

5 The volume, V, of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.

The volume of a sphere is increasing at a constant rate of $24\,\text{cm}^3\,\text{s}^{-1}$. Find the rate of increase of the radius when the radius is 6 cm. [4]

6 **(a)** The position vectors of the points P, Q and R relative to an origin O are $\begin{pmatrix}4\\7\end{pmatrix}$, $\begin{pmatrix}8\\5\end{pmatrix}$ and $\begin{pmatrix}x\\y\end{pmatrix}$ respectively. The point R lies on PQ extended such that $3\overrightarrow{QR} = 2\overrightarrow{PR}$. Use a vector method to find the values of x and y. [3]

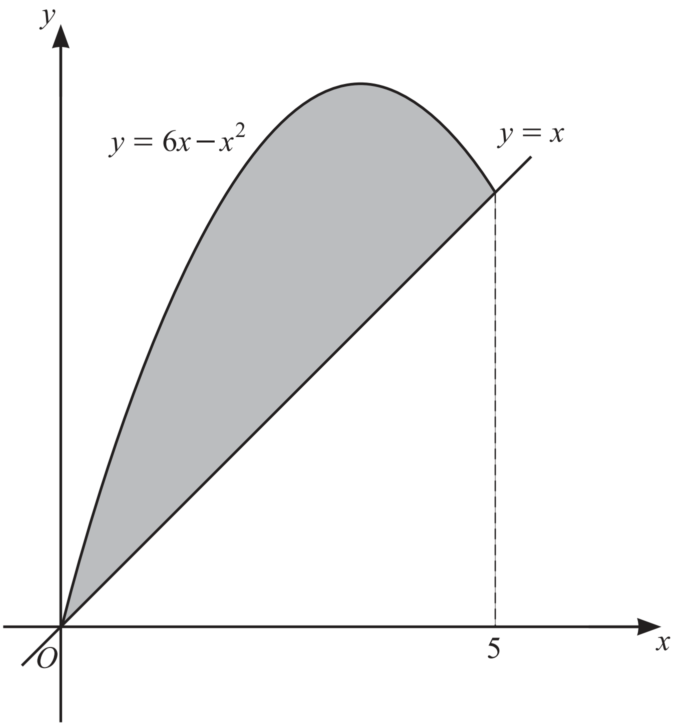
(b) You are given that **i** is a unit vector due east and **j** is a unit vector due north.

Three vectors, **a**, **b** and **c** are in the same horizontal plane as **i** and **j** and are such that $\mathbf{a}+\mathbf{b} = \mathbf{c}$.
The magnitude and bearing of **a** are 5 and 210°.
The magnitude and bearing of **c** are 10 and 330°.

(i) Find **a** and **c** in terms of **i** and **j**. [2]

(ii) Find the magnitude and bearing of **b**. [5]

7 **(a)**

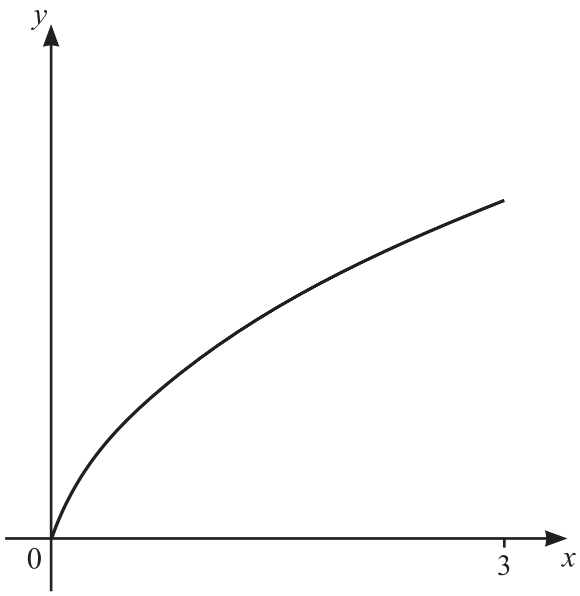


The diagram shows the curve $y = 6x - x^2$ for $0 \leqslant x \leqslant 5$ and the line $y = x$. Find the area of the shaded region. [4]

(b) **(i)** Find $\int\left(\frac{1}{(2x-6)^3}+\cos x\right)\mathrm{d}x$. [3]

(ii) Find $\int\frac{(x^4+1)^2}{2x}\mathrm{d}x$. [3]

8 **(a)**



The diagram shows the graph of $y = \mathrm{f}(x)$ where f is defined by $\mathrm{f}(x) = \dfrac{3x}{\sqrt{5x+1}}$ for $0 \leqslant x \leqslant 3$.

(i) Given that f is a one-one function, find the domain and range of f^{-1}. [3]

(ii) Solve the equation $\mathrm{f}(x) = x$. [2]

(iii) On the diagram above, sketch the graph of $y = \mathrm{f}^{-1}(x)$. [2]

(b) The functions g and h are defined by

$$g(x) = \sqrt[3]{8x^3 + 3} \quad \text{for} \quad x \geqslant 1,$$

$$h(x) = e^{4x} \quad \text{for} \quad x \geqslant k.$$

(i) Find an expression for $g^{-1}(x)$. [2]

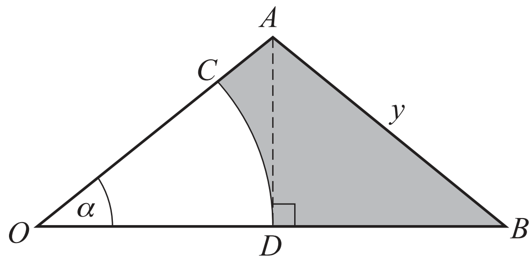
(ii) State the least value of the constant k such that gh(x) can be formed. [1]

(iii) Find and simplify an expression for gh(x). [1]

9 In this question all lengths are in centimetres and all angles are in radians.

(a) The area of a sector of a circle of radius 24 is $432\,\text{cm}^2$. Find the length of the arc of the sector. [4]

(b)



The diagram shows an isosceles triangle, *OAB*, with $AO = AB = y$ and height *AD*.
OCD is a sector of the circle with centre *O*. Angle *AOB* is α.

(i) Find an expression for *OB* in terms of y and α. [1]

(ii) Hence show that the area of the shaded region can be written as $\frac{y^2}{2}\cos\alpha(2\sin\alpha - \alpha\cos\alpha)$. [3]

10 In the expansion of $\left(ax+\frac{b}{x^2}\right)^9$, where a and b are constants with $a > 0$, the term independent of x is $-145\,152$ and the coefficient of x^6 is -6912. Show that $a^2b = -12$ and find the value of a and the value of b. [7]

Question 11 is printed on the next page.

11 The line with equation $x+3y=k$, where k is a positive constant, is a tangent to the curve with equation $x^2+y^2+2y-9=0$. Find the value of k and hence find the coordinates of the point where the line touches the curve. [9]